Proposal to Roof the Courtyards of an Historical Building in Pisa with Glass and Steel Grid Shells: Form Finding and Stability Problems

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Abstract: The technical note describes a proposal to roof two internal courtyards of the main building of the Engineering Faculty at the University of Pisa by means of thin flat shells made of a quadrilateral mesh of steel bars supporting plane glass plates stiffened by a grid of steel cables. An exposure of the structural design concepts and of the geometrical genesis of the shells is presented accompanied by a stability analysis and a description of the technological solutions found to solve some crucial constructional problems.

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Introduction

The historical seat of the Engineering Faculty at the University of Pisa is an interesting example of early 20th century masonry architecture, located at few steps from the cathedral square, Piazza dei Miracoli (see Figs. 1 and 2).

The building has three internal rectangular courtyards presently roofless and subutilized. Under the increasing demand from the students for more lecture and socialization spaces, the Faculty Direction recently decided to promote, within the graduating students of Civil Engineering, engineerized architectonical ideas on how to cover the yards with a very light and transparent roof characterized by the minimal visual and structural impact imposed by the particular location of the faculty building which did not allow any strong change of its external appearance.

The new roof had therefore to remain almost invisible from the outside and at the same time, in order to obtain a minimum interference with the old building, it was also not to be supported along the perimeter of each yard, but just on a few points of it.

The present paper synthesizes the final graduate thesis of the first author which best matched the initial requests and which has been awarded for the year 2006 with the annual prize of the Italian Association of Steel Builders ACAI (Del Guerra et al. 2006) (see Fig. 3).

Form Finding

These restricting geometrical requests and the necessity to obtain at the same time structural efficiency, suggested covering each one of the two quadrilateral courtyards with double curved shells supported along the four lateral sides by tubular steel arches connected to the masonry only at the four corners of the courtyard.

As is well known, shells always played an important role in the progress of bearing structures, particularly as continuous reinforced concrete surfaces, since double curvature allows membrane structural response, thus implying very thin and elegant surfaces. Many spectacular and cost-efficient examples have been realized in the past, such as Torroja’s Fronton Recoletos or Musumeci’s Ponte sul Basento (see Fig. 4), but in the last decades of the twentieth century the growing costs of forms and scaffolding has lowered the interest in this kind of structure with just a few exceptions such as the works of Isler and Diete (Anderson 2004).

Recently, interest has been shifted to continuous reticulated surfaces as assemblages of single prefabricated linear elements. Many examples show that single layer grid shells are reticulated spatial structures that can be used for light and elegant constructions (Schlaich 2002; Baldassini and Menerat 2004; Schlaich et al. 2001). Besides the advantage of being prefabricated and sometimes preassembled in a shop, grid structures allow for a higher architectural freedom, given the fact that the spaces between the structural elements can be covered with different kinds of panels, including transparent glass panes.

Triangular and Quadrilateral Meshes

The design of the mesh is one of the most important points to be taken into account when designing single layer grid shells, because it has a great influence both on the appearance and on the costs of the structure. Triangular or quadrilateral meshes can be used. The triangular mesh can be easily adapted to any kind of geometry of the surface and has the further advantage of being stiff in the middle plane of the surface. The main disadvantage is that triangular covering panels are statically inefficient with reference to out of plane forces at the corners, this meaning additional costs with glass panels. Moreover, each node requires six connections instead of the four of the quadrilateral mesh.
In the present project a quadrilateral mesh was then chosen, which requires simpler connections and thinner glass panels. When opting for a quadrilateral mesh, we have to consider first that the mesh is much less stiff in the plane with rapport to the triangular one and even hypostatic if pinned at the corner of the mesh. Second, should we use plane covering panels, we could not always adapt them to a free-form surface, since a plane contains only three of the four corners, while the fourth one belongs to the same plane only if the geometry is defined accordingly.

**Generation of a Surface Made of Plane Quadrilateral Facets**

Costs minimization can be achieved when the structural elements have equal length all through the surface. Such geometrical uniformity is immediately obtained with a quadrilateral mesh supporting plane quadrilateral panels in cylindrical, single curved surfaces.

Double curved, sinclastic surfaces can be covered by plane quadrilateral facets only if the surface generation follows strict rules.

A rotational surface can in general be subdivided into planar quadrilateral facets, but the length of the elements cannot be the same throughout the surface although the rotational symmetry permits to have groups of identical elements.

To fulfill the equal length requirement, it was chosen to generate the surface starting from two identical arcs, respectively, contained in two vertical, mutually orthogonal planes disposed along the diagonals of a square figure, 18 m side, concentric to the courtyard (see Fig. 5).

The first arc (directrix curve) was translated along the other arc (ruling curve) always maintaining the orthogonality of the respective planes. Each arc, with a 25.5 m span and 4.9 m rise, was then subdivided in 22 segments of equal length (1.27 m).

In this way, it was obtained a sinclastic translational surface covering a 25.5 m × 25.5 m square whose diagonals are parallel to the sides of the rectangular courtyard underneath.

Finally, the surface was delimited laterally by intersecting it with four inclined planes, each of them comprising a set of nodes, thus obtaining at the borders four planar curves.

The roofing surface is connected to the existing building at the corners by means of reticulated plane steel frames whose inclination was chosen to allow the best transmission of membranes forces.
Constructional Detailing

Steel Grid
The grid shell is composed of 600 steel bar elements with rectangular $60 \times 40$ mm section. Of which 550 are perfectly identical and the other 50 are of two different types, for a total of only three different bar typologies. They are all connected to each other by the same connection detail: in each node converge four bars and a couple of cables, the bolt connection allowing for a rotation between the two direction of elements in the tangent plane of the surface, while out of plane the connection maintains bending stiffness (see Fig. 6). The steel used is Fe510 (nominal value of the yield strength $f_y = 335$ MPa). Between the middle plane of the bars and the plane of the cables there is a small 40 mm eccentricity.

Steel Arches and the Connections to the Masonry
Along the borders, four arcs with circular hollow sections stiffen and reinforce the roof surface. These tubular arcs are shop assembled with all the connection plates so that, in situ, only eight weldings are necessary while all the other connections are bolted. These arcs are connected to the masonry by means of four reticulated steel structure, each one connected to a reinforced concrete slab in correspondence of each of the four corners of the courtyard by means of a cylindrical bearing (see Fig. 7). All the beams are shorter than 12 m, easy to be transported, galvanized, and lifted. The steel used for the arcs is Fe430 (nominal value of the yield strength $f_y = 275$ MPa).

Open Sides
The lateral sides are partially closed with glass panes suspended by means of rotules. A clearance of approximately 100 mm is left between the panes and the existing roof, in a way that an air change between the courtyard and the exterior is maintained (see Fig. 8).

The new roof overlaps the existing one, and the drainage is left to the existing drainage system. It is accepted that under the most severe conditions of rain and wind—very few times in a year—the internal space of the courtyard is not completely watertight.

The subvertical elements that support the lateral glass panes are designed in a way that they do not stiffen the tubular arcs, otherwise the membrane structural response would be impaired.

Stability Problems

Nonlinear Behavior: Global and Local Instability Analyses
The high span/rise ratio of the shell and the presence of the cables, reacting just to tensile stresses, are responsible for the mechanical nonlinear behavior of the structure (Baldassini and Raynaud 2004) requiring deformations to be taken into account in calculating the forces in the elements. Indeed, sinclastic surfaces predominantly submitted to compression efforts exhibit a typical softening, nonlinear behavior where internal forces grow more than proportionally to the applied loads.

Nonlinear analysis (see Fig. 9) was also necessary to estimate the minimal critical load relative to local and global instability phenomena. Actually, when a structure presents a softening behavior, its rigidity diminishes with the growing intensity of the loads, and eventually might reduce to zero, in which case the structure collapses. This snap-through instability is very dangerous because it might be quasi-instantaneous and releases a great amount of energy so that a local snap-through instability may induce a global collapse. In the presence of softening behavior the
Eurocodes [Eurocode 3 (2005)] require global security factors higher than those of the usual ultimate limit state (ULS), but their magnitude is not specified and left to the experience and sensibility of the designer. It is then important to verify the structure under loads multiplied for the ULS combination coefficient and for a further “α_cr” coefficient greater than 1 and here chosen equal to 1.75 (where α_cr is the ratio collapse load/ULS combination load).

**Sensitivity of the Structure to Imperfections**

Another crucial point is the sensitivity of the structure to imperfections. In fact, particular distribution of initial geometrical or imposed imperfections, always present in a real structure, might consistently diminish the collapse load, and it is then necessary to take them into account.

For each loading combination were considered the deformations \( \delta_n = (\delta_x, \delta_y, \delta_z) \) of the structure at the nodes of the initial structural model, not affected by imperfections. New structural models were created, one for each loading combination, in which the geometry of the nodes was updated with distributions of geometrical imperfections proportional to the deformations of the structure \( \delta_n = (x_n, y_n, z_n) \). The α_cr coefficient was calculated as the maximum value before numerical instability under the increasing load. This procedure was repeated for different values of maximal vertical imperfection \( \max \beta(\delta_n) = 25, 50, 75, 100 \text{ mm} \), thus obtaining α- versus-displacement graphs (where “α” is the ratio applied load/ULS load), which give a clear idea of the safety resources of the structure (see Fig. 10). The two most important parameters are the α_cr coefficient and the residual stiffness of the structure when α is equal to 1.

Among the load combinations considered, the most severe were those combining the asymmetric bitriangular Eurocode 1 snow load distribution with permanent loads and wind pressure loads.

**Stiffening Role of the Cables**

In a structure of this kind a significant role is played by the cables. The quadrilateral mesh of the single layer grid shell is much less stiff than the triangular one and, if pinned at the corners, even hypostatic and not able to transfer the in-plane shear forces at all. Disposing a couple of cables along the diagonals, we guarantee a very efficient way of carrying the shear forces. The cables are very thin (8 mm diameter) and thus not perceptible from underneath, thus allowing for a very minimal, filigrane structure. Prestressing the cables permits having a better transmission of the efforts. A prestressing value of 15 kN for each cable was chosen as a good compromise between structural behavior and constructional requirements.

Cables are also used as chords to the four bearing supports that connect the structure to the masonry building. The main role of these cables is to absorb horizontal forces caused by permanent loads and to prevent the masonry structure to be charged with such damaging actions. A parametric study has shown that thanks to those chords the steel structure is not sensitive to the masonry building lateral thrust stiffness. This has led to a proposed revised solution where only one of the four bearings is fixed, the others left free to slide on the horizontal plane; in this way, the existing building would be charged only by vertical forces (approximately...
440 kN of total permanent load, a relatively low load compared to the self-weight of the masonry structure, but flashing and waterproofing systems would be more complicated in the area of the sliding bearings.

**Conclusions**

Roofing courtyards in historical buildings impose a strong integration process between the basic architectonical idea, aiming at obtaining an almost immaterial surface, and the structural engineering conception since geometrical aspects as well as structural efficiency are both of primary importance and highly interacting.

Reticulated shells made of steel bars and glass panes constitute an elegant solution to this problem. Thanks to their high transparency and lightness they are less invasive than other more traditional covering constructions. Quadrilateral meshes allow for highly industrialized and economic transformation processes, and structural efficiency may be achieved by introducing a set of diagonal prestressed cables.

**References**


